A new non-perturbative time-dependent string configuration

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Abstract

A time-dependent bosonic string configuration is discussed, in graviton and dilaton backgrounds, leading to Weyl-symmetry beta-functions which are homogeneous in X^0, to any order in α'. As a consequence, a string reparametrization can always be implemented, such that beta functions can be cancelled, to any order in α'. This non-perturbative conformal invariance is valid for any target space dimension, and leads to a power law expanding Universe, for which the power vanishes if a specific relation between the dimension and dilaton amplitude holds. Finally, $D = 4$ is the minimum dimension (in the case of a spherical world sheet) for which this configuration is consistent with a Wick rotation in a Minkowski target space.

The usual approach in String Cosmology consists in cancelling one-loop (or two-loop) beta functions corresponding to Weyl invariance, and solving the corresponding differential equations in order to find the relevant backgrounds. An alternative approach is discussed here [1], where, instead of cancelling perturbative orders of the beta functions, we look for a configuration of the bosonic string leading to beta functions which are homogeneous functions of X^0, to all orders in α'. This property will be enough to recover Weyl invariance, as the two-loop and higher orders in α' of the beta functions are not fixed in a unique way, but depend on the renormalization scheme which is used when computing the trace of the energy-momentum tensor of the string [2]. Changing renormalization scheme corresponds to reparametrizing the string couplings, which in our case will not affect the time-dependence of the configuration, but will only rescale the metric by a constant and add another constant to the dilaton.

In order to find a hint on what configuration can lead to homogeneous beta functions, we first present an exact functional method, dealing with the sigma model provided by the string action, where the evolution of the quantum theory with the amplitude of α' is derived. We will see that, if the target space metric is conformally flat and proportional to the second derivative of the dilaton (in the string frame), the latter is independent of the amplitude of quantum fluctuations. Homogeneous beta functions will then be obtained for a specific case, where the graviton is a power law of X^0.

We consider a bosonic string on a spherical world sheet, with bare action

$$S = \frac{1}{4\pi}\int d^2\xi\sqrt{\gamma}\left\{\gamma^{ab}\lambda\eta_{\mu\nu}\partial_a X^\mu\partial_b X^\nu + R^{(2)}\phi_{bare}(X^0)\right\}, \tag{1}$$

where $\lambda = 1/\alpha^{'}$ is left as a free parameter which controls the amplitude of quantum fluctuations. $\phi_{bare}(X^0)$ is a bare dilaton, whose precise form is not relevant, but it should in principle contain at least a cubic term in X^0, in order to generate quantum fluctuations. The quantum theory is obtained by defining the connected graphs generator functional and its Legendre transform Γ, for a given value of λ. We obtain then a family of quantum theories, labelled by λ and described by Γ_λ. The exact evolution equation for Γ with λ was derived in [1], and leads to the following evolution for the dilaton in the quantum theory (no summation on i: g_{ii} is any of the space components of the metric):

$$
\begin{aligned}
g_{00}\dot{\phi} &= -\frac{\Lambda^2}{8\pi R^{(2)}}\left(1+(D-1)\frac{g_{00}}{g_{ii}}\right) \\
&\quad +\frac{\phi^{''}}{4g_{00}}\ln\left(1+\frac{2\Lambda^2 g_{00}}{R^{(2)}\phi^{''}}\right),
\end{aligned} \tag{2}
$$

where a dot represents a derivative with respect to λ and a prime a derivative with respect to X^0. Λ is a *fixed* world sheet cut off, and we stress here the difference with a Wilsonian approach, where $\lambda = 1/\alpha^{'}$ would be fixed and the world sheet cut off would be running. As a consistency check, one can easily see that a linear dilaton and flat metric [3] lead to $\dot{\phi} = 0$ (after a rescaling of the space coordinates $X^i \to X^i\sqrt{D-1}$), which is expected, as such a configuration does not generate quantum fluctuations. There is another possibility, thought, to obtain an $\alpha^{'}$-independent dilaton: from the evolution equation (2), if g_{00} is proportional to g_{ii} and to $\phi^{''}$, it is always possible to obtain $\dot{\phi} = 0$ by rescaling the space coordinates X^i (this rescaling absorbs the world sheet cut off). As a consequence, a necessary condition for the non-trivial solution to be independent of the amplitude of quantum fluctuations is that

$$
g_{\mu\nu}(X^0) \propto \phi^{''}(X^0)\ \eta_{\mu\nu}. \tag{3}
$$

We now turn to the conformal properties of a configuration satisfying the condition (3). We first note that the one-loop beta functions corresponding to Weyl invariance cannot be cancelled by a configuration satisfying the condition (3). But, for the following specific case

$$
\begin{aligned}
g_{\mu\nu}(X^0) &= \frac{\kappa}{(X^0)^2}\eta_{\mu\nu} \\
\phi(X^0) &= \phi_0 \ln X^0,
\end{aligned} \tag{4}
$$

where κ and ϕ_0 are constants, the different terms in the one-loop beta functions happen to be homogeneous to the same power of X^0. We checked that the next order in $\alpha^{'}$ again contains homogeneous functions of X^0, which are also homogeneous to the first order. Finally, this property is valid to any order in $\alpha^{'}$: whatever power of the Ricci or Riemann tensor is considered, and multiplied by covariant derivatives of the dilaton, contracting the indices with the metric or the inverse metric will always lead to the same power of X^0. As a consequence, we have for the configuration (4)

$$
\beta^g_{00} = \frac{1}{(X^0)^2}\sum_{m=1}^{\infty}\xi_m\left(\frac{\alpha^{'}}{\kappa}\right)^m,
$$

$$
\begin{aligned}
\beta^g_{ij} &= \frac{\delta_{ij}}{(X^0)^2}\sum_{m=1}^{\infty}\zeta_m\left(\frac{\alpha'}{\kappa}\right)^m ,\\
\beta^\phi &= \frac{1}{\alpha'}\sum_{m=1}^{\infty}\eta_m\left(\frac{\alpha'}{\kappa}\right)^m ,
\end{aligned}
\tag{5}
$$

where ξ_n, ζ_n, η_n are coefficients independent of α'.
The next step is to argue that, from two-loops and above, the coefficients ξ_n, ζ_n, η_n are not unique but depend on the renormalization scheme which is used, in order to calculate the trace of the energy momentum tensor of the string [2]. A change of renormalization scheme corresponds to a reparametrization of the string, which leaves the S matrix invariant, and reads, at two loops

$$
\begin{aligned}
\tilde{g}_{\mu\nu} &= g_{\mu\nu} + \alpha' g_{\mu\nu}\left(b_1 R + b_2\partial^\rho\phi\partial_\rho\phi + b_3\nabla^2\phi\right),\\
\tilde{\phi} &= \phi + \alpha'\left(c_1 R + c_2\partial^\rho\phi\partial_\rho\phi + c_3\nabla^2\phi\right),
\end{aligned}
\tag{6}
$$

where $b_1, ..., c_1, ...$ are any constants. In the specific case of the configuration (4), this reparametrization just rescales the metric by a constant factor and adds another constant to the dilaton. But the important point is that the reparametrization (6) changes the beta functions in the following way [2]

$$
\tilde{\beta}^i = \beta^i + (\tilde{g}^j - g^j)\frac{\partial\beta^i}{\partial g^j} - \beta^j\frac{\partial}{\partial g^j}(\tilde{g}^i - g^i), \tag{7}
$$

such that it is always possible to choose the constants $b_1, ..., c_1, ...$ to cancel the two-loop beta functions $\tilde{\beta}^i$, what was done explicitly in [1]. We then conjecture that the configuration (4) satisfies Weyl invariance to any order in α', since the reparametrization (6) can be extended to any order, and the modification (7) is always valid. This cancellation obtained after a reparametrization of the string is possible for any target space dimension, and for any dilaton amplitude.

Concerning Wilsonian properties of the configuration (4), it was shown in [1] that the latter is an infrared fixed point of momentum flows defined on the world sheet. To show this, an exact renormalization equation was derived, following the approach given in [4], where a sharp cut off was used (this is indeed enough if one considers the evolution for the potential part only of the Wilsonian action). As a consequence, the α'-fixed point solution of the equation (2) was identified with a world sheet Wilsonian IR fixed point.

In order to find the cosmological properties of the configuration (4), we remind that the metrics in the string frame and the Einstein frame are related by

$$
dt^2 - a^2(t)(d\vec{x})^2 = \exp\left(\frac{-4\phi(x^0)}{D-2}\right) g_{\mu\nu}(x^0)dx^\mu dx^\nu, \tag{8}
$$

where t is the cosmic time and $a(t)$ is the scale factor of the corresponding spatially flat FRW Universe. Plugging the configuration (4) into the identity (8), we obtain the cosmic time as a power law of x^0, and the scale factor is

$$
a(t) = a_0 t^{1+\frac{D-2}{2\phi_0}}, \tag{9}
$$

where a_0 is a constant. We obtain then a power-law expanding Universe, whose power vanishes if the following relation holds

$$D - 2 + 2\phi_0 = 0, \tag{10}$$

which is therefore the condition to have a Minkowski target space. Note that, if expressed in terms of the cosmic time, the dilaton is independent of ϕ_0 and reads

$$\phi = -\frac{D-2}{2} \ln t. \tag{11}$$

To conclude with the configuration (4), we discuss its properties under a target space Wick rotation. Up to now, the world sheet metric had a Euclidean signature, but the target space metric had a Minkowski signature. Because of the logarithmic dilaton, the analytic continuation $X^0 \to iX^0$ generates an imaginary part in the action. As a consequence, the partition function acquires a phase and becomes, in the case of a spherical world sheet,

$$Z \to Z_E \times \exp(i\pi\phi_0), \tag{12}$$

where Z_E is the Euclidean partition function (for both world sheet and target space metrics). In order to have a real partition function, we need ϕ_0 to be an integer. Taking in addition the condition (10) to have a Minkowski target space, we find that the allowed dimensions are all the even integers, starting from 4:

$$D = 4, 6, 8, \cdots \tag{13}$$

$D = 4$ is thus the minimum dimension where the Wick rotation in target space is consistent with the configuration (4).

As a last remark, one can notice that the logarithmic dilaton, in X^0, leads to a diverging Euclidean partition function, for late times, in the case $D = 4$ and $\phi_0 = -1$, for which the target space is flat and static. This feature shows that a UV cut off in time is necessary for the consistency of the whole picture.

Finally, the extension of this work to a superstring would not change the results, as the homogeneity of the beta functions corresponding to Weyl invariance would not be affected.

References

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